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Vaccine Mathematics, Modelling and Manufacturing

IRC Distinguished Lecture Series

Numerical Stability Analysis for Delay Differential and Renewal Equations

Dr. Dimitri Breda, University of Udine

August 27-29, 2018, 10:30-11:30am

LIAM, Kinsmen 227, 8 Chimneystack Road, York University

Delay equations of differential and renewal type are always more appropriate to model complex phenomena in a more realistic way, thanks to the presence of delayed terms relating the current evolution to the past history. Population dynamics, ecology and epidemics are among a broad range of areas where delays arise naturally. The price to pay for this more realistic mathematical description is that time delays lead to dynamical systems on infinite-dimensional state spaces. The use of pseudospectral methods to reduce delay equations to finite dimension systems is reviewed in this series of three lectures, with the goal of addressing fundamental questions about stability and bifurcation.

Lecture 1: Stability of equilibria: the infinitesimal generator approach
Lecture 2: Stability of periodic solutions: the solution operator approach
Lecture 3: Bifurcation of nonlinear problems: back to ODEs

The NSERC/Sanofi Distinguished Lecture speaker **Dr. Dimitri Breda** received the Laurea degree summa cum laude in Mechanical Engineering from the University of Udine in 1998 and the Ph.D. degree in Computational Mathematics from the University of Padova in 2004. He is an Associate Professor of Numerical Analysis at the Department of Mathematics, Computer Science and Physics, University of Udine. He is also the vice-coordinator of the PhD program in Computer Science, Mathematics and Physics and head of the Computational Dynamics Laboratory – CDLab (cdlab.uniud.it). He is a leading researcher in the field of numerical and applied mathematical analysis, in particular, in numerical methods for the stability analysis of infinite-dimensional dynamical systems generated by delay and other functional equations, with applications to real-life problems ranging from population dynamics to control theory.

LIAM LABORATORY for Industrial & Applied MATHEMATICS

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Lecture 1: Stability of equilibria: the infinitesimal generator approach

Even though the dynamics is posed on an infinite-dimensional state space, still much of the properties of ODEs hold. The celebrated principle of linearized stability for equilibria is an example: hyperbolic rest points of nonlinear models inherit the stability features of the corresponding linearizations. The latter are dictated by the spectrum of the infinitesimal generator of the associated semigroup of solution operators. The pseudospectral approach is used to reduce the generator to a matrix, whose eigenvalues converge rather fast to the exact ones: the so-called characteristic roots. The method is presented for delay differential equations, and extension to renewal equations is briefly accounted for.

Lecture 2: Stability of periodic solutions: the solution operator approach

The principle of linearized stability holds also for periodic orbits of delay differential equations, for which indeed a Floquet theory is available. The same cannot be said for renewal equations. Nevertheless, monodromy operators can be defined and their spectrum can be efficiently approximated via another pseudospectral approach, this time discretizing the evolution semigroup. The method is presented for renewal equations, illustrating also the main differences occurring when dealing with delay differential equations. Beyond the main use to compute Floquet multipliers, the technique can be extended to estimate Lyapunov exponents to possibly detect chaotic behaviors.

Lecture 3: Bifurcation of nonlinear problems: back to ODEs

The infinitesimal generator approach discussed in the first lecture can be extended to nonlinear problems. Then one ends up with an approximating system made of a finite number of ODEs. On the one hand, the structure of this system is rather peculiar and can be conveniently exploited. On the other hand, efficient and consolidated software is available for continuation of ODEs, so that in principle one is able to perform a broad, yet approximated bifurcation analysis. For many realistic models this approach represents the only chance so far to investigate the relevant dynamics. In this lecture the method is illustrated for coupled delay differential and renewal equations, which are often the base of recent models of physiologically structured populations.

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